

Anomalous Gluon Production and Condensation in Glasma

Aiichi Iwazaki

*International Economics and Politics, Nishogakusha University,
6-16 3-bantyo Tiyoda Tokyo 102-8336, Japan.*

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The collinear color electric and magnetic fields have been discussed to be produced immediately after high energy heavy ion collisions. We discuss anomalous gluon production under the background gauge fields. The gluons are Nielsen-Olesen unstable modes. The production rate of the modes by Schwinger mechanism has recently been found to be anomalously larger than the rate of quarks or other stable gluons. Analyzing classical evolutions of the modes with initial conditions given by vacuum fluctuations, we find that their production makes the color electric field decay very rapidly. The life time of the field is approximately given by the inverse of saturation momentum in the collisions. We discuss a possibility that the mode with zero momentum form a Bose condensate and its gluon number density grows to be of the order of $1/\alpha_g$. The condensate would melts into quark gluon plasma after the gluon number density grows sufficiently large for nonlinear interactions to be operative.

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I. INTRODUCTION

It has recently been paid attention to that color electric (E) and magnetic (B) fields produced in high energy heavy ion collisions[1] decays by producing quarks and gluons. Namely, glasma decays into quark gluon plasma (QGP). One of the most efficient driving forces for the decay is the Schwinger mechanism[2–4]. The production rate of massless quarks by the mechanism is similar to the production rate of gluons. The difference simply comes from the difference in the color charges, spins and statistics. However, when the color magnetic field (B) is present, the production rate of the gluons becomes anomalously larger than the production rate of the quarks. This important fact has recently been shown by Itakura and Tanji[5]. They have found that the production rate of the gluons is proportional to $\exp(\pi B/E)$, while the rate of the quarks is proportional to $\exp(-\pi B/E)$ or unity.

As is well known, the gluons can have imaginary energies under the color magnetic field. Namely, the states with magnetic moments parallel to \vec{B} in the lowest Landau level are unstable; their energies ($\propto \sqrt{p_3^2 - gB}$) are imaginary when $p_3^2 < gB$, where p_3 is a momentum component in the direction parallel to $\vec{B} = (0, 0, B)$ (g is a gauge coupling constant). Thus, their amplitudes exponentially grow with time. The amplitudes of the other states of the gluons simply oscillate with time, because their energies are real. The presence of such unstable modes (Nielsen-Olesen unstable modes[6]) has prevented us to find a proper quantization of the gluons. Hence, the production of the Nielsen-Olesen (NO) unstable gluons by the Schwinger mechanism was not discussed. But the above authors pointed out that the unstable modes become stable owing to the acceleration by the electric field. This is because the momentum $p_3(t) = \int^t dt' gE$ becomes large sufficiently such that the imaginary energy becomes real with $p_3^2(t) > (gB)^2$. Consequently they have derived the production rate of the NO gluons.

The importance of their result is that the production rate, in other words, the occupation number ($\propto \exp(\pi B/E)$) of a gluon state associated with the NO unstable gluons is much larger than the unity. This should be compared with the typical occupation number ($\propto \exp(-\pi B/E)$) associated with other stable gluons or massless quarks[3]. It is much less than the unity. The large production rate of the gluons leads to the rapid decay of the electric field. In particular, the decay process accelerates because the production rate $\propto \exp(\pi B/E)$ increases more as the electric field E becomes weaker. Hence, when we discuss the decay, it is important to include the back reaction of the electric field to the production of the gluons. Furthermore, the fact of the large occupation number allows us to classically analyze the production of the NO gluons. When we classically analyze the production, we can easily take into account the back reaction of the electric field to the gluon production. The back reaction has not yet been discussed by the above authors[5].

In this paper, by assuming that the NO modes are initially produced by vacuum fluctuations, we analyze the classical evolution of the modes and the color electric field. We find that the electric field rapidly decays owing to the acceleration of the large amount of the NO gluons. The life time of the electric field is approximately given by Q_s^{-1} where Q_s denotes saturation momentum in high energy heavy ion collisions. We show that the color electric current carried by the gluons is much larger than that of the quarks produced by the Schwinger mechanism. Thus, the

influence of the quarks on the decay of the electric field is negligible compared with that of the gluons. We also discuss that the NO gluons form a Bose condensate, because of the large occupation number of the gluons. Especially, the amplitude of the NO mode with zero momentum is the largest and grows most rapidly. It can grow to be of the order of $1/g$ until nonlinear interactions of gluons are operative. Then, it would melt into QGP through the equipartition of the momentum by the nonlinear interactions. It implies that the number density of the gluon condensate grows up to of the order of $1/g^2$.

In the next section II we briefly review the Nielsen-Olesen unstable modes and discuss anomalous production of the modes. In the section III, we discuss field configurations of the Nielsen-Olesen unstable modes and find basic equations governing the temporal behaviors of the modes as well as the color electric field. In the section IV, we find that the gluons of the modes are dominantly produced and their production leads to the rapid decays of the electric field. In the section V we discuss the gluon condensation of the NO modes, which may arise before the thermalization of the quark gluon plasma. In the final section VI we summarize our results.

II. NIELSEN-OLESEN UNSTABLE MODES

We first explain our formalism and briefly review the NO unstable modes. We also explain the anomalous production of the NO modes under the electric field. We consider SU(2) gauge theory and suppose the background color electric and magnetic fields given by $\vec{E}_a = \delta_{a,3}(0, 0, E)$ and $\vec{B}_a = \delta_{a,3}(0, 0, B)$. They are supposed to be spatially homogeneous and collinear both in the real and color spaces. The gauge fields are represented by the gauge potential $A_\mu \equiv A_\mu^{a=3}$. Under the background fields, the gauge potentials $\Phi_\mu \equiv (A_\mu^1 + iA_\mu^2)/\sqrt{2}$ perpendicular to A_μ^3 behave as charged vector fields. When we represent SU(2) gauge potentials A_μ^a using the variables A_μ and Φ_μ , Lagrangian of SU(2) gauge potentials is written in the following,

$$L = -\frac{1}{4}F_{\mu,\nu}^2 - \frac{1}{2}|D_\mu\Phi_\nu - D_\nu\Phi_\mu|^2 - ig(\partial_\mu A_\nu - \partial_\nu A_\mu)\Phi^\dagger{}^\mu\Phi^\nu + \frac{g^2}{4}(\Phi_\mu^\dagger\Phi_\nu - \Phi_\nu^\dagger\Phi_\mu)^2, \quad (1)$$

with $F_{\mu,\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $D_\mu = \partial_\mu - igA_\mu$, where we used a gauge $D_\mu\Phi^\mu = 0$. Obviously, we find that the fields Φ_μ represent charged vector fields with the anomalous magnetic moment described by the term $-ig(\partial_\mu A_\nu - \partial_\nu A_\mu)\Phi^\dagger{}^\mu\Phi^\nu$. Therefore, it is easy to see that when the magnetic field $B = \partial_1 A_2 - \partial_2 A_1$ is present, but $E = 0$, the particles represented by the fields Φ occupy the Landau levels and interact with each other through the term $\frac{g^2}{4}(\Phi_\mu^\dagger\Phi_\nu - \Phi_\nu^\dagger\Phi_\mu)^2$. The energies of the states in the Landau levels denoted by integer $N \geq 0$ are given by $E_N = \sqrt{2gB(N + 1/2) \pm 2gB + p_3^2}$, where \pm denotes magnetic moment parallel (-) or anti-parallel (+) to \vec{B} .

Among them we notice the states in the lowest Landau level ($N = 0$) with the magnetic moment parallel to \vec{B} . Their energies can be imaginary; $E_{N=0} = \sqrt{p_3^2 - gB}$. Thus, the modes with the imaginary energies exponentially increase or decrease with time. That is, the field Φ representing the modes evolves with time such that $\Phi \propto \exp(-iE_{N=0}t) = \exp(\pm|E_{N=0}|t) = \exp(\pm|\sqrt{gB - p_3^2}|t)$. The states are called as Nielsen-Olesen unstable modes. Among them, the mode with $p_3 = 0$ increases or decreases most rapidly. The presence of such unstable modes implies the instability of the vacuum state, that is, the state with the background color magnetic field B . This is similar to the case that the state $\psi = 0$ is unstable in a model of a complex scalar field with the double well potential $-m^2|\psi|^2 + \frac{\lambda}{2}|\psi|^4$ ($m^2 > 0$). In this model unstable modes around the state $\psi = 0$ exist and exponentially grow such as $\psi \propto \exp(t\sqrt{m^2 - \vec{p}^2})$ where \vec{p} denotes a momentum. Similarly the background gauge fields with color magnetic fields are unstable. Indeed, the classical simulations[7] have been performed to show the instability of the background gauge fields involving color electric and magnetic fields. The instability in the simulation has been discussed[8, 9] to be caused by the NO unstable modes: Their amplitudes exponentially grow and then saturate when nonlinear interactions are operative owing to the growth of the amplitudes.

The spontaneous production[10] of the unstable modes (or gluons) is caused by the color magnetic field. Thus, it is not the Schwinger mechanism. When we analyze the Schwinger mechanism of the NO modes, the presence of the imaginary energy was an obstacle because we cannot properly quantize the modes. But, we should note that in order to obtain the production rate by the Schwinger mechanism[3], we only need in-state (in infinite past) and out-state (in infinite future) under the electric field. Although the NO modes are unstable, they are stable in the infinite past and future in the presence of the electric field. This is because the square of the momentum $p_3(t) = \int^t dt' gE$ is sufficiently large in the past and future such as $p_3^2(t) - gB > 0$. The stability of the NO modes in the infinite past and future allows us to estimate the production rate[5] of the modes under the electric field.

In general, charged fields oscillate with the frequency $\propto gEt$ in the past ($t \rightarrow -\infty$) and future ($t \rightarrow +\infty$) under the electric field E , owing to the acceleration by the electric field. The frequency is real and depends on time. In

this case the production rate of the fields is less than unity; it is proportional to, for example, $\exp(-\pi B/E)$ when the magnetic field is present. But, in the case of the NO modes they pass a period in which their frequency becomes imaginary so that their amplitudes exponentially grow. Before and after the period they simply oscillate with the real frequency. In other words, the modes oscillate and their amplitudes smoothly change in the far past, but once they enter the unstable period, their amplitudes exponentially grow with time. After passing the period, they oscillate again but with much larger amplitudes than those before passing the period. These behaviors are peculiar to the NO modes. In particular, the exponential growth of the amplitudes leads to the anomalous production rate $\propto \exp(\pi B/E)$.

We may understand a naive physical reason why the rate increases more as the electric field becomes weaker. The large production rate comes from the fact that the modes pass the period in which they exponentially grow. They stay the period approximately for the time $\Delta t = \sqrt{gB}/gE$ because the momentum increases within the time such that $\Delta p_3 = \Delta t gE = \sqrt{gB}$. Hence, the amplitude grows by $\exp(\sqrt{gB}\Delta t) = \exp(B/E)$. Weaker electric field causes the longer stay in the unstable period and hence the larger growth of the amplitude. Although it is a rough estimation, it explains why the production rate of the gluons $\propto \exp(\pi B/E)$ becomes large as the electric field becomes weak.

Hereafter we take only the NO modes and analyze their production under the electric field. The NO modes are described by the field $\Phi \equiv (\Phi_1 + i\Phi_2)/\sqrt{2}$ and are governed by the following Hamiltonian

$$H = \int d^3x \left(\frac{1}{2}(\partial_0 A_3)^2 + |\partial_0 \Phi|^2 + |(\vec{\partial} - ig\vec{A})\Phi|^2 - 2gB|\Phi|^2 \right) = \int d^3x \left(\frac{1}{2}(\partial_0 A_3)^2 + |\partial_0 \Phi|^2 + |(\partial_3 - igA_3)\Phi|^2 - gB|\Phi|^2 \right), \quad (2)$$

where we neglected the nonlinear interactions $\frac{g^2}{4}(\Phi_\mu^\dagger \Phi_\nu - \Phi_\nu^\dagger \Phi_\mu)^2$ in eq(1). The color electric and magnetic fields are given such that $\vec{E} = \partial_0 \vec{A} = (0, 0, \partial_0 A_3)$ and $\vec{B} = \vec{\partial} \times \vec{A} = (0, 0, B)$. The nonlinear interactions are not operative as long as the amplitude Φ is small. When the field Φ grows large such that the nonlinear interactions are operative, the NO modes couple with the other modes in higher Landau levels as well as themselves.

We note that the last term in the Hamiltonian represents a negative potential. When the magnetic field forms a flux tube, the term represents a negative potential with finite width given by the width of the flux tube. Whether or not the field Φ possesses unstable modes depends on the existence of the states trapped in the negative potential.

It is interesting to see the analogy between our model and the model of the complex scalar field with the double well potential mentioned above. The model describes Cooper pairs condensates. Thus, the decay of the color electric field corresponds to the decay of an external electric field imposed on superconductors. The question is how fast the external electric field decays, just after a metal in a normal state is supercooled with a temperature below the critical temperature separating superconducting ($\langle \psi \rangle = \sqrt{m^2/\lambda}$) and the normal states ($\langle \psi \rangle = 0$). The normal state $\psi = 0$ decays by producing the Cooper pairs ψ , which condense to form the state $\langle \psi \rangle = \sqrt{m^2/\lambda}$. Since they are accelerated by the electric field, the electric field loses its energy and vanishes.

III. PRODUCTION OF NIELSEN-OLESEN UNSTABLE MODES

Because the production rate is much larger than unity (this implies that the occupation number in a state is much larger than unity), the production of the NO gluons represented by NO modes may be classically analyzed. Then, we can easily take into account the back reaction of the electric field to the gluon production. In this section we will formulate basic equations governing the back reaction.

First, we discuss the assumption that the background gauge fields are homogeneous in the transverse plane perpendicular to the collinear fields \vec{B} and \vec{E} . When the NO unstable modes are excited, they destroy the homogeneity because of the localization of the wave functions of the modes.

$$\phi \equiv (x_1 - ix_2)^n \exp\left(-\frac{gB|z|^2}{4} + ip_3 x_3\right), \quad (3)$$

with $z \equiv x_1 + ix_2$ and integer $n \geq 0$ where we used a gauge potential $\vec{A} = (-Bx_2/2, Bx_1/2, 0)$. The effect of the back reaction induces the inhomogeneity in the electric field; the currents carried by the modes are not homogeneous so that the electric field affected by the currents is also not homogeneous.

But, by taking the appropriate linear combination of the NO unstable modes we can form almost homogeneous field configurations in the transverse plane. Then, their currents are also almost homogeneous. Such field configurations are given by,

$$\Phi = \sum_{i=1 \sim N} \phi_i(\vec{x}), \quad \phi_i(\vec{x}) = \int dp_3 c(p_3) \exp\left(-\frac{gB|z-z_i|^2}{4} + ip_3 x_3\right), \quad (4)$$

with $c(p_3)$ is a dimensionless function of the longitudinal momentum p_3 and $z_i = x_{1,i} + ix_{2,i}$, where each component ϕ_i satisfies the condition, $\phi_i \phi_j \simeq \delta_{i,j} \phi_i^2$ since we impose that $|z_i - z_j| \geq l_B \equiv \frac{1}{\sqrt{gB}}$. Namely, a configuration ϕ_i is separated with the nearest neighbors approximately by the distance l_B . Furthermore, we assume that the area L^2 of the transverse plane is given by $L^2 = Nl_B^2$. Thus, we find that the field configuration Φ is approximately uniform in the transverse space. This kind of the configuration of the NO modes was analyzed[11] to discuss so called "spaghetti vacuum".

Using the field configuration, we rewrite the Hamiltonian of the NO unstable modes,

$$H = \int d^3x \left(\frac{1}{2}(\partial_0 A)^2 + |\partial_0 \Phi|^2 + |(i\partial_3 - gA)\Phi|^2 - gB|\Phi|^2 \right) \simeq N \int d^3x \left(\frac{1}{2}(\partial_0 A)^2 + |\partial_0 \phi|^2 + |(i\partial_3 - gA)\phi|^2 - gB|\phi|^2 \right), \quad (5)$$

with $\phi = \int dp_3 c(p_3) \exp(ip_3 x_3 - gB|z|^2/4)$, where the color electric field is given by $E = \partial_0 A$ with the homogeneous gauge potential $A \equiv A_3$. The Hamiltonian describes the NO modes under the homogeneous background electric and magnetic fields. The first term represents the energy of the electric field and the other terms represent the energy of the NO modes. We can see that the last term with the magnetic field gB represents a negative potential for the NO modes ϕ . Thus, it gives rise to the imaginary energy of the field $\phi \propto \exp(-i\epsilon t)$ with $\epsilon^2 = -gB < 0$.

If the magnetic field forms a flux tube with a finite width, it gives a negative potential with the finite width. Thus, if the field is trapped by the potential, the energy ϵ can be imaginary, but its absolute value is smaller than \sqrt{gB} ; it depends on the width of the tube. When the width is infinite, the energy ϵ is given by $\sqrt{-gB}$. On the other hand, when the width is finite, as it becomes smaller, the absolute value of energy ϵ becomes smaller. The flux tubes of the background gauge fields are more realistic field configurations produced in high energy heavy ion collisions than the homogeneous ones under consideration. Since the gauge fields are homogeneous in the longitudinal direction, they can be viewed as an ensemble of electric and magnetic flux tubes with various widths. Based on the view, we can understand[8, 9] the results of the numerical simulations[7]. Although the flux tubes are realistic one, it is meaningful to analyze the anomalous gluon production in the homogeneous gauge fields in order to see physical essences of the production.

We proceed to analyze the back reaction. For the purpose, we decompose the field $\phi(\vec{x})$ into the components of the momentum eigenstate,

$$\phi = \frac{1}{\sqrt{L^3}} \sum_{\vec{p}} \phi_p \exp(i\vec{p}\vec{x}) \quad (6)$$

with

$$\phi_p = \frac{8\pi^2 c(p_3)}{gBL^{3/2}} \exp\left(-\frac{p_T^2}{gB}\right) \quad (7)$$

where the transverse momentum p_T is given such that $p_T^2 \equiv p_1^2 + p_2^2$. Then, it follows that

$$H = L^2 \left(\frac{L}{2}(\partial_0 A)^2 + \frac{1}{l_B^2} \sum_{\vec{p}} (|\partial_0 \phi_p|^2 + |(p_3 + gA)\phi_p|^2 - gB|\phi_p|^2) \right), \quad (8)$$

where we have used the following formulas,

$$\int d^3x \exp(i\vec{p}\vec{x}) = (2\pi)^3 \delta^3(p) = L^3 \delta_{p,0}^3. \quad (9)$$

The equations of motions of the fields ϕ_p and the gauge field A are given by respectively

$$\partial_0^2 \phi_p = gB\phi_p - (p_3 + gA)^2 \phi_p \quad \text{and} \quad L\partial_0^2 A = -\frac{2g}{l_B^2} \sum_{\vec{p}} (p_3 + gA) |\phi_p|^2 \quad (10)$$

where the second equation represents a Maxwell equation $\partial_0 E = -J$ with the current $J = \frac{2g}{l_B^2} \sum_{\vec{p}} (p_3 + gA) |\phi_p|^2$. It describes how the electric field changes by the effect of the current J .

We should point out that the largest amplitude of the NO modes is given by the mode with the vanishing transverse and longitudinal momentum, that is, the mode with $p_T = 0$ in eq(7) and $p_3 + gA = 0$ in eq(10). The mode grows most rapidly compared with the other modes with $p_3 + gA \neq 0$. Here, the momentum $p_3 + gA(t) = p_3 + g \int_0^t dt' E(t')$ is the momentum gained by the mode with initial momentum p_3 owing to the acceleration by the electric field E . We naively expect from the similarity to the model of the complex scalar field that the mode with zero momentum form a stable Bose condensate. But as we discuss later, although the mode form a Bose condensate with its amplitude of the order of $1/g$, the condensate is unstable.

In order to solve the equations we need to impose initial conditions. The initial condition of the electric field is given such that $E(t=0) = \partial_0 A(t=0) = E_0$ and $A(t=0) = 0$. This corresponds to the fact that we switch on the electric field $E = E_0$ at $t=0$. In other words, we consider the situation that high energy heavy ion collisions occur at $t=0$ and the coherent color gauge fields are produced at the instance.

How should we choose initial conditions of the field ϕ ? Before the collisions, there are no color electric and magnetic fields. The gluons with small x form color glass condensates in nuclei. Just after the collisions the gluons form the coherent gauge fields E and B , but there is no classical field ϕ . Thus, we may naively choose the initial conditions such that $\partial_0 \phi(t=0) = 0$ and $\phi(t=0) = 0$. But these initial conditions lead to the trivial result; $\phi(t) = 0$ for any time $t > 0$. Therefore we need to find an appropriate initial condition. As we explained in the previous section, the NO modes are spontaneously generated owing to the instability of the state with the homogeneous magnetic field. Thus, it is reasonable to take the initial conditions given by the vacuum fluctuations,

$$\phi_p(t=0) \equiv \sqrt{\langle \hat{\phi}_p^2 \rangle} \quad \text{and} \quad \partial_t \phi_p(t=0) \equiv \sqrt{\langle (\partial_t \hat{\phi}_p)^2 \rangle} \quad (11)$$

where $\hat{\phi}_p$ denotes the momentum component of the free massless scalar field $\hat{\phi}$ with no background fields. The state $|\rangle$ represents the vacuum state $\langle \hat{\phi} \rangle = 0$. Then, the vacuum can be represented by the following wave functionals

$$\Psi(\phi_p) \propto \exp\left(-\sum_{\vec{p}} |p| |\phi_p|^2\right) \quad \text{and} \quad \Psi(\partial_0 \phi_p) \propto \exp\left(-\sum_{\vec{p}} \frac{|\partial_0 \phi_p|^2}{|p|}\right), \quad (12)$$

with $|p| \equiv \sqrt{p_T^2 + p_3^2}$. Namely, we assume that the initial conditions are given by the vacuum fluctuations in the vacuum without E and B . Then, using the wave functionals, we can calculate the expectation values in eq.(11). As the NO modes are given such that $\phi_p = \frac{a(p_3)}{gBL^{3/2}} \exp(-\frac{p_T^2}{gB})$ with arbitrary dimensionless function $a(p_3) \equiv 8\pi^2 c(p_3)$, we insert this function into the wave functionals and obtain the distributions of $a(p_3)$ and $\partial_0 a(p_3)$ in the vacuum,

$$\Psi(a(p_3)) \propto \exp\left(-\sum_{p_3} \frac{|a(p_3)|^2 T(p_3 l_B)}{4\pi L \sqrt{gB}}\right) \quad \text{and} \quad \Psi(\partial_0 a(p_3)) \propto \exp\left(-\sum_{p_3} \frac{|\partial_0 a(p_3)|^2 U(p_3 l_B)}{4\pi L (gB)^{3/2}}\right) \quad (13)$$

with $l_B \equiv 1/\sqrt{gB}$, where the functions $T(p_3 l_B)$ and $U(p_3 l_B)$ are given by

$$T(x) \equiv \int_0^\infty dy \sqrt{y+x^2} \exp(-2y) \quad \text{and} \quad U(x) \equiv \int_0^\infty dy \frac{\exp(-2y)}{\sqrt{y+x^2}}. \quad (14)$$

Using the wave functionals we calculate the initial conditions for ϕ_p or $a(p_3)$. Obviously the distribution of the real part of $a(p_3)$, that is, ϕ_p is identical to that of the imaginary part of $a(p_3)$. The expectation values of these components can be obtained in the following,

$$\langle a(p_3)^2 \rangle = \frac{\int_{-\infty}^{+\infty} dz z^2 g(z, p_3)}{\int_{-\infty}^{+\infty} dz g(z, p_3)} = \frac{\pi \sqrt{gB} L}{T(p_3 l_B)} \quad \text{and} \quad \langle (\partial_0 a(p_3))^2 \rangle = \frac{\int_{-\infty}^{+\infty} dz z^2 f(z, p_3)}{\int_{-\infty}^{+\infty} dz f(z, p_3)} = \frac{\pi (gB)^{3/2} L}{U(p_3 l_B)} \quad (15)$$

with

$$g(z, p_3) \equiv \exp\left(-\frac{z^2 T(p_3 l_B)}{4\pi\sqrt{gBL}}\right) \quad \text{and} \quad f(z, p_3) \equiv \exp\left(-\frac{z^2 U(p_3 l_B)}{4\pi(gB)^{3/2}L}\right). \quad (16)$$

We rewrite the equations of motion using the variables of the real or imaginary part of $a(p_3)$ instead of ϕ_p ,

$$\partial_0^2 b(p_3) = gBb(p_3) - (p_3 + gA)^2 b(p_3) \quad \text{and} \quad L\partial_0^2 A = -\frac{g}{4\pi L} \sum_{p_3} (p_3 + gA) b(p_3)^2 \quad (17)$$

where the real function $b(p_3)$ represents the real or imaginary part of $a(p_3)$. Therefore, the initial conditions are given by

$$A(t=0) = 0, \quad \partial_0 A(t=0) = E_0 \quad \text{and} \quad b(p_3, t=0) = \sqrt{\frac{\pi\sqrt{gBL}}{T(p_3 l_B)}}, \quad \partial_0 b(p_3, t=0) = \sqrt{\frac{\pi(gB)^{3/2}L}{U(p_3 l_B)}}. \quad (18)$$

Because these equations involve the size L of the system, it apparently seems that our results depend on the size L . But as we show below, our results, e.g. life time of the electric field, are independent on the size L . In order to see the independence, we further rewrite the equations of motion by using the variable $\alpha(p_3) \equiv \sqrt{\frac{T(p_3 l_B)}{\pi\sqrt{gBL}}} b(p_3)$,

$$\partial_0^2 \alpha(p_3) = gB \alpha(p_3) - (p_3 + gA)^2 \alpha(p_3) \quad \text{and} \quad \partial_0^2 gA = -\frac{g^2 \sqrt{gB}}{4\pi} \int_{-\infty}^{+\infty} dp_3 \frac{(p_3 + gA) \alpha(p_3)^2}{T(p_3 l_B)} \quad (19)$$

with the initial conditions $\alpha(p_3, t=0) = 1$ and $\partial_0 \alpha(p_3, t=0) = \sqrt{\frac{gBT(p_3 l_B)}{U(p_3 l_B)}}$, where we used the relation $\sum_{p_3} = \frac{L}{2\pi} \int_{-\infty}^{+\infty} dp_3$. Obviously both the equations of motion and initial conditions are independent on the system size L . Furthermore, the color electric current J is given by

$$J = \frac{g\sqrt{gB}}{4\pi} \int_{-\infty}^{+\infty} dp_3 \frac{(p_3 + gA) \alpha(p_3)^2}{T(p_3 l_B)}, \quad (20)$$

which is also independent on L . The current vanishes at $t=0$ because $T(x) = T(-x)$ and $A(t=0) = 0$. Using these equations of motion, we can discuss the temporal behaviors of the electric field $\partial_0 A$ and the NO modes $\alpha(p_3)$ by taking into account the back reaction of the electric field to the production of the modes.

For the convenience, we write down the field ϕ of the NO modes in terms of the variable α ,

$$\phi = \frac{1}{\sqrt{L^3}} \sum_{\vec{p}} \phi_p \exp(i\vec{p}\vec{x}) = \frac{1}{\sqrt{L^3}} \sum_{\vec{p}} \frac{(\alpha(p_3) + i\alpha(p_3))\sqrt{\pi}}{(gB)^{3/4}L\sqrt{T(p_3 l_B)}} \exp\left(-\frac{p_T^2}{gB}\right) \exp(i\vec{p}\vec{x}), \quad (21)$$

where $\alpha(p_3)$ is a dimensionless real function. We can see that the vacuum fluctuation $\phi(t=0)$ of the NO modes is of the order of unity compared with the background gauge fields $\sim O(1/g)$.

IV. NUMERICAL RESULTS

Now we wish to discuss the production of the NO gluons and quarks. Especially, we would like to discuss the ratio between the amount of the gluons and that of the quarks produced by the electric field. In order to discuss the amounts of the particles we compare the color electric current of the quarks with that of the gluons. We show that the amount of the gluons is about a hundred times larger than that of the quarks. As a result the color electric field rapidly decays owing to this anomalous gluon production.

First we derive a relevant equation describing the production of the quarks by the Schwinger mechanism. The equation has been previously explored[4]. I would like to explain it shortly. We assume that the quarks are massless and they form a SU(2) doublet. Then, the charges of the quarks are given by $g/2$ and $-g/2$. Both of them possess

their anti-particles with their charges given by $-g/2$ and $g/2$, respectively. Therefore, we have four massless fermions; a pair of the quarks in a $SU(2)$ doublet and their anti-quarks. They are two fermions with the charge $g/2$ and two fermions with the charge $-g/2$. Their number densities are identical to each other because a pair of a positive and a negative charged fermions is created at the same moment under the electric field.

It has been recently shown[4, 12] that the evolution of the number density n_q of the massless fermions is governed by the chiral anomaly when collinear strong electric and magnetic fields are present. In particular the anomaly equation is very efficient when the magnetic field is sufficiently strong such that the particles in the production are only allowed to occupy the lowest energy states in the lowest Landau level. Then, the equation of the chiral anomaly is given by

$$\partial_0 J_0^5 = 4\partial_0 n_q = 2 \frac{(g/2)^2 E(t) B}{2\pi^2}, \quad (22)$$

where we assumed the homogeneity of the chiral current in the transverse and longitudinal direction; $\vec{\partial} \vec{J}^5 = 0$. The equality $\partial_0 J_0^5 = 4\partial_0 n_q$ in eq(22) comes from the fact that all of the four fermions have the positive chirality when \vec{E} parallel to \vec{B} . This is because the positive (negative) charged fermions are accelerated to the direction parallel (antiparallel) to \vec{E} and their spins point to the direction (antiparallel) parallel to \vec{B} when they occupy the lowest Landau level.

Obviously, the chiral anomaly describes how the number density n_q evolves with time under the effect of the electric and magnetic fields. The electric field loses its energy owing to the acceleration of the quarks as well as the gluons. Hence, we add the contribution of the quarks to the Maxwell equation. Consequently, the equations describing the evolution of the number of the quarks and the NO modes as well as the evolution of the electric field are given by

$$\partial_0 n_q = \frac{g^2 E(t) B}{16\pi^2}, \quad \partial_0^2 \alpha(p_3) = gB \alpha(p_3) - (p_3 + gA)^2 \alpha(p_3) \quad \text{and} \quad \partial_0^2 gA = -4g^2 n_q - \frac{g^2 \sqrt{gB}}{4\pi} \int_{-\infty}^{+\infty} dp_3 \frac{(p_3 + gA) \alpha(p_3)^2}{T(p_3 l_B)}, \quad (23)$$

where $4gn_q$ denotes the current of the massless quarks. The initial conditions are given by

$$n_q(t=0) = 0, \quad \alpha(p_3, t=0) = 1, \quad \partial_0 \alpha(p_3, t=0) = \sqrt{\frac{gB T(p_3 l_B)}{U(p_3 l_B)}}, \quad A(t=0) = 0 \quad \text{and} \quad \partial_0 A(t=0) = E_0. \quad (24)$$

By solving these equations we can see how the electric field vanishes owing to the production of the NO modes. Furthermore, we can obtain the temporal behaviors of the electric current densities of the quarks $J_q = 4gn_q$ and the gluons $J_g = \frac{g\sqrt{gB}}{4\pi} \int_{-\infty}^{+\infty} dp_3 \frac{(p_3 + gA) \alpha(p_3)^2}{T(p_3 l_B)}$. Both of them vanish at $t = 0$. After the electric field is switched on at $t = 0$, the pair production of the quarks arises and their electric current flows. Similarly, the gluons as the NO modes are produced in the vacuum fluctuations and their electric current flows along the electric field. Owing to the production of the quarks and the gluons, the electric field decreases and vanishes at $t = t_c > 0$. Hence we compare the electric current of the quarks with that of the gluons at the time $t = t_c$ when the electric field vanishes,

$$R(t = t_c) = \frac{J_g}{J_q} = \frac{\frac{g\sqrt{gB}}{4\pi} \int_{-\infty}^{+\infty} dp_3 \frac{(p_3 + gA(t=t_c)) \alpha(p_3, t=t_c)^2}{T(p_3 l_B)}}{4gn_q(t = t_c)}, \quad (25)$$

where all of the integration range of the momentum p_3 is taken. But because the relevant modes we should take into account are the NO unstable modes, the integration range should be limited to the range in which each mode $\alpha(p_3, t)$ passes the unstable period until the electric field vanishes. The range is approximately given such that $|p_3 + gE_0 t_c| < \sqrt{gB}$ for $p_3 < 0$ and $p_3 < \sqrt{gB}$ for $p_3 > 0$. As our analysis of the gluons is classical, our estimation of the ratio is not rigorous. Thus, rigorously speaking, we do not know the appropriate integration range: If we take all of the integration range, we take into account irrelevant modes to NO instability, which do not pass the unstable period. Therefore, we may take the integration range $|p_3| < \sqrt{gB}$ in order to see roughly how large amount of the gluons are produced compared with that of the quarks.

In Fig.1 we show the temporal behaviors of the electric field and the ratio R with the parameters $g = B = E_0 = 1$. We have taken the integration range $|p_3| < \sqrt{gB}$ and checked that the result does not change even if the integration range $|p_3| < 1.2\sqrt{gB}$ is taken. We find that when the electric field vanishes, the electric current of the gluons produced is approximately eighty times larger than that of the quarks. Owing to this fact, the life time of the electric field is much shorter than the life time in the case of the decay only by the production of the quarks. Actually, the above

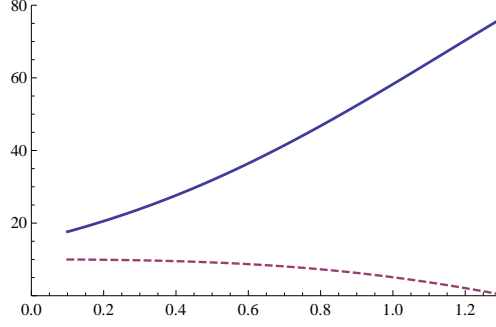


FIG. 1: temporal behaviors of electric field $10E(t)$ (dashed line) and the ratio $R(t)$ (solid line)

equations can be explicitly solved when the contribution of the gluons is neglected, i.e. $\alpha = 0$. The solution of the electric field is given by

$$E = E_0 \cos\left(\frac{\sqrt{g^3 B} t}{2\pi}\right). \quad (26)$$

The solution represents a plasma oscillation[3, 4]. Hence, the life time when E vanishes is given by $\pi^2/\sqrt{g^3 B}$, which is equal to $\pi^2 \sim 10$ with $g = B = 1$. It is roughly 8 times longer than the life time shown in the figure. The life times are given by $1.3Q_s^{-1}$ and $10Q_s^{-1}$ respectively in the physical unit Q_s^{-1} ; Q_s ($= 1\text{GeV} \sim 2\text{GeV}$) denotes saturation momentum of high energy heavy ion collisions in RHIC or LHC. In this way the decay of the electric field is mainly caused by the anomalous gluon production, that is, the production of the NO unstable modes. The contribution of the quarks is negligible.

The NO modes are generated by the vacuum fluctuations and are amplified by the magnetic field. At the same time, they are accelerated by the electric field. In Fig.2 we show the temporal behavior of the momentum distribution ϕ_p or $\alpha(p_3, t)/\sqrt{T(p_3 l_B)}$, that is, the growth of the amplitude with time. We can see that the vacuum fluctuations (shown by the curve at $t = 0$ in Fig.2) give the momentum distribution symmetric in p_3 with the peak at $p_3 = 0$. The peak moves to points with negative momentum with time. This is because the gluons are accelerated by the electric field so that the momentum with which the amplitude $\alpha(p_3, t)/\sqrt{T(p_3 l_B)}$ has the largest growth rate is given by $p_3 = -\int_0^t dt' gE(t') = -gA(t) < 0$.

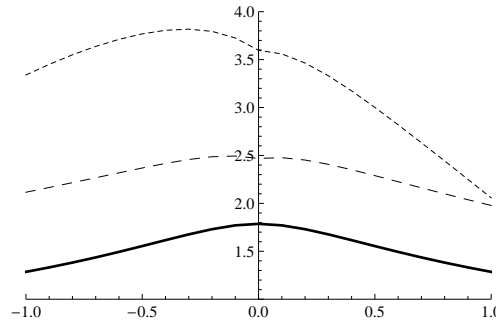


FIG. 2: amplitudes $\alpha(p_3, t)/\sqrt{T(p_3 l_B)}$ at $t = 0$ (solid line), $t = 0.5$ (dashed line) and $t = 1.0$ (short dashed line)

We can see, however, from the figure that the peak is not located at $p_3 = -\int_0^t dt' gE(t')$, but at momentum near $p_3 = 0$ as long as t is small. This is because the initial condition $\frac{\partial_0 \alpha(p_3, t=0)}{\sqrt{T(p_3 l_B)}} = \sqrt{\frac{gB}{U(p_3 l_B)}}$ becomes larger, as $|p_3|$ becomes larger. Thus, as long as t small, the initial condition dominates the growth rate of the modes. Indeed, at $t < 0.5$ the maximum of $\phi_p \propto \alpha(p_3, t)/\sqrt{T(p_3 l_B)}$ is given by $\phi_p(p_3 \simeq 0)$. However, it approaches the value $\phi_p(p_3 = -\int_0^t dt' gE(t'))$ as t goes beyond 0.5. The effect of the initial condition diminishes with time.

V. GLUON CONDENSATION

We have mentioned the analogy between the NO unstable modes and unstable modes in the model of the complex scalar field. The model describes Cooper pairs in superconductors. Obviously, the unstable modes with zero momentum in the model form a Bose condensate $\langle\psi\rangle = \sqrt{m^2/\lambda}$ of the Cooper pairs. That is, once the modes are produced, the amplitudes of the modes exponentially grow to the value of the stable state. They are stabilized by the nonlinear interaction $\lambda|\psi|^4$. Hence, it is natural to expect from the similarity that the gluons of the NO modes may form a Bose condensate[13].

The NO modes typically carry the momentum $\vec{p} = (p_T \simeq \sqrt{gB}, p_3 \simeq \sqrt{gB})$. This is because the modes have the spatially transverse extension given by $1/\sqrt{gB}$ and their longitudinal momentum p_3 given by $\Delta p_3 = -gE\Delta t$ with the acceleration by the electric field within the time $\Delta t = \sqrt{gB}/gE$. Among them, the mode with the vanishing transverse and longitudinal momenta ($p_T = 0$ and $p_3 + gA = 0$), has the largest amplitude and growth rate. Furthermore, the amplitude of the mode can grow to of the order of $1/g$ until the nonlinear interactions among the NO modes are operative, as we will show below. Hence, we may think that the mode form a Bose condensate of gluons with zero momentum. But, contrary to the expectation from the similarity to the model of the complex scalar field, the nonlinear interactions in the gauge theory do not stabilize the exponential increase of the mode. They are three and four point functions of the gauge fields. The interactions cause the momentum transfer from the mode to the other stable modes in higher Landau levels or they produce new NO modes under new magnetic fields produced by the electric current of the old NO modes, etc. (see our comment below). These new modes induced by the interactions have larger momenta than those of the old NO modes. That is, the interactions make the modes with higher momenta rapidly increase. In general the number density of the gluons is proportional to the square of the amplitude of the gluon field ϕ . Hence, once the nonlinear interactions are operative, the number of gluons with larger momenta increases, while the number of the gluons in the condensate decreases. In this process the number of gluons is not conserved.

We can estimate how large the mode with zero momentum grows until the nonlinear interactions are operative. Namely, we estimate the maximum number of the gluons with zero momentum in the condensate. We note that the nonlinear interactions are given schematically by $g^2\Phi^4$ or $gA\Phi^2$ in eq(1); the background gauge fields A are assumed to be of the order of $1/g$. They are smaller than the kinetic terms of Φ and A_μ when the amplitude Φ is much less than of the order of $1/g$. But when $\Phi \sim 1/g$, all of the interaction terms become of the same order of the magnitude as the kinetic terms. Hence, the amplitude of the mode with zero momentum can grow to of the order of $1/g$, in other words, the number density of the gluons becomes of the order of $1/g^2$. The most of gluons composing the background electric field become the gluons in the condensate. In this way, the gluon condensate with zero momentum arises on the way to the thermalization of QGP.

Here we wish to make a comment on the numerical simulation[7] by Berges et al.. We can see in the simulation that after the NO modes as the primary instability grow large, the secondary instability arises. This secondary instability has been discussed[9] to arise owing to the excitation of new NO unstable modes under azimuthal magnetic field, which is produced by the electric current J carried by the NO modes. Namely, the electric current J of the NO modes becomes large as the modes grow so that the azimuthal magnetic field B_T induced by J becomes strong. When B_T becomes strong enough, new NO unstable modes are induced. These new NO modes lead to the secondary instability and possess larger momenta ($p_3 > \sqrt{gB}$) than momenta ($p_3 < \sqrt{gB}$) of the NO modes leading to the primary instability. Obviously, the secondary instability arises owing to the nonlinear interactions[9]; Thus, the appearance of the secondary instability would be onset of the thermalization of the gluons. Actually, the growth rate of the secondary instability is much larger than that of the primary instability. Furthermore, the growth rate becomes larger as the momentum $|p_3|$ of the modes is larger, while the growth rate of the primary instability becomes larger as $|p_3|$ is smaller. These facts implies that after the gluon condensate with the number density of the gluons of the order of $1/g$ is formed, the rapid momentum transfer arises from the modes with small momentum ($< \sqrt{gB}$) to the modes with large momentum ($> \sqrt{gB}$). That is, the number of the gluons with the large momenta rapidly increases owing to the effects of the nonlinear interactions. The process would lead to the equipartition of the momentum and eventually thermalization of QGP. This simulation indicates that the gluon condensate may arise before the thermalization.

It is interesting to speculate that inverse cascades, with which the modes with small momenta are amplified in a model of scalar fields shown in the recent paper[14], correspond to the excitations of the NO modes and cascades, with which the modes with large momenta are amplified, correspond to the excitation of the modes of the secondary instability. We need to analyze more closely the correspondence in order to confirm this speculation.

VI. SUMMARY AND DISCUSSION

Motivated by the recent study of the anomalous production of the Nielsen-Olesen unstable modes by the Schwinger mechanism, we have discussed the decay of the color electric field in the classical approximation by taking into account the back reaction of the electric field to the gluon and quark production. We have found that the electric field rapidly decays owing to the anomalous production of the gluons. The contribution of the quarks to the decay is negligible. We have also found that the amount of the produced gluons is about a hundred times larger than that of the quarks.

A model of color glass condensate predicts that the color electric and magnetic fields are produced immediately after high energy heavy ion collisions. Fluid dynamical simulations of thermalized QGP suggest that they should decay into the plasma within a time 1 fm/c. Our analysis indicates that such a very fast decay is caused by the anomalous gluon production. Actually, our analysis shows that the decay is completed within the time of the order of Q_s^{-1} .

We have also discussed a possibility that the gluons of the NO unstable mode form a Bose condensate with zero momentum until the nonlinear interactions in the gauge theory are operative. The number of the gluons in the condensate rapidly increases and then saturated when it becomes of the order of $1/g^2$. After the condensate with such a large number of the gluons is formed, the nonlinear interactions lead to the rapid increase of the number of gluons with large momentum ($> \sqrt{gB}$). Hence the equipartition of the momentum and eventually the thermalization of QGP would be achieved.

We have discussed the decay of the color electric field in glasma. The color magnetic field in glasma also decays in the following. In general the longitudinal color magnetic fields form flux tubes, which expands with time. Owing to the expansion, electric field δE_T perpendicular to the magnetic field is induced according to the Faraday's law of induction. On the other hand, owing to the expansion of the longitudinal electric flux tubes, a magnetic field δB_T is induced, which is parallel or anti-parallel to the electric field δE_T . Thus, under the field δB_T , NO modes $\delta\phi_T$ are excited and make the electric field δE_T decay rapidly. Eventually, the expansion of the magnetic flux tube induces the electric field δE_T , which decays by the acceleration of the NO modes $\delta\phi_T$. This implies the decay of the magnetic flux tube. This is the decay mechanism of the magnetic field.

We have discussed the gluon production in the non-expanding glasma. When we treat it in the expanding glasma, the similar analysis is possible. But, we should use the initial conditions for the NO modes shown in the recent paper[15]. Using the initial conditions, we can properly take into account quantum effects of the modes under the background gauge fields.

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